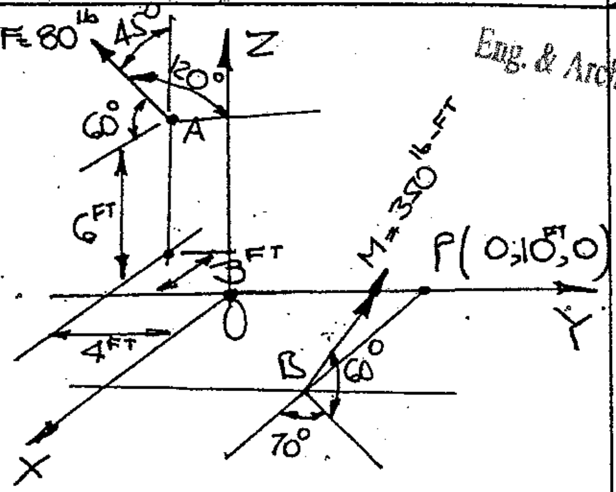
PROBLEM 4_129



POSITION VECTOR @ P:

$$\overrightarrow{OP} = 0\overrightarrow{I} + 10\overrightarrow{J} + 0\overrightarrow{K}$$

UNIT VECTOR @ B ALONG M:

$$\overrightarrow{U_B} = (\cos 60)(\cos 70)\overrightarrow{I} + (\cos 60)(\cos 20)\overrightarrow{J} + (\sin 60)\overrightarrow{K} = 0.171\overrightarrow{I} + 0.470\overrightarrow{J} + 0.866\overrightarrow{K}$$

UNIT VECTOR @ A ALONG F:

$$\overrightarrow{U_A} = (\cos 60)\overrightarrow{I} + (\cos 120)\overrightarrow{J} + (\cos 45)\overrightarrow{K}$$
$$= 0.5\overrightarrow{I} + (-0.5)\overrightarrow{J} + (0.707)\overrightarrow{K}$$

POSITION VECTOR @ A:

$$\overrightarrow{OA} = -3\overrightarrow{I} + (-4)\overrightarrow{J} + 6\overrightarrow{K} \qquad \overrightarrow{PA} = \overrightarrow{OA} - \overrightarrow{OP} = -3\overrightarrow{I} - 14\overrightarrow{J} + 6\overrightarrow{K}$$

EQUILIBRIUM @ P OF FORCES REQUIRES $\underline{\overrightarrow{F_P} = \overrightarrow{F} = 80^{lb}\,\overrightarrow{U_A}}$

EQUILIBRIUM @ P OF MOMENTS REQUIRES $\overrightarrow{M_P} = \overrightarrow{M} + \overrightarrow{PA} \times \overrightarrow{F}$

$$\overrightarrow{M_P} = 350^{lb}(\overrightarrow{U_B}) + \overrightarrow{PA} \times (80^{lb}\overrightarrow{U_A})$$

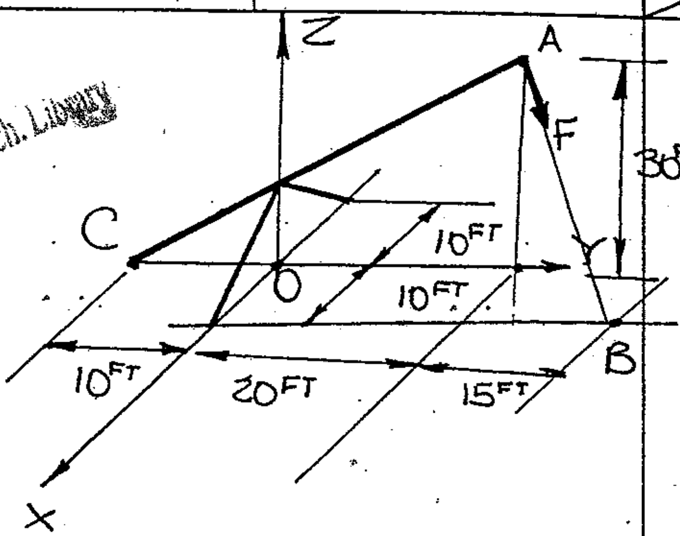
$$= 350^{lb}(0.171\overrightarrow{I} + 0.470\overrightarrow{J} + 0.866\overrightarrow{K}) + 80^{lb}\begin{vmatrix} \overrightarrow{I} & \overrightarrow{J} & \overrightarrow{K} \\ -3 & -14 & 6 \\ 0.5 & -0.5 & 0.707 \end{vmatrix}$$

$$= 59.85\overrightarrow{I} + 164.5\overrightarrow{J} + 303.1\overrightarrow{K} + \left(-409.68\overrightarrow{I} - 551.84\overrightarrow{I} + 680\overrightarrow{K}\right)$$

$$\underline{\overrightarrow{M_P} = -492\overrightarrow{I} + 574.2\overrightarrow{J} + 983.1\overrightarrow{K}} \ \ (\text{lb-FT})$$

$$\underline{\overrightarrow{F_P} = 40\overrightarrow{I} - 40\overrightarrow{J} + 56.6\overrightarrow{K}}$$

PROBLEM 4_130



$F = 50^{lb}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$\overrightarrow{CA} = \overrightarrow{OA} - \overrightarrow{OC}$

EQUIVALENT SYSTEM AT POINT C:

1- FORCE AT C $\vec{F_C} = \vec{F} = (50^{lb}\ \vec{U_A})$ WHERE $\vec{U_A}$ IS UNIT VECTOR AT A ALONG $\vec{F}$.

$$\vec{U_A} = \frac{\vec{OB} - \vec{OA}}{AB} = \frac{10\vec{I} + 15\vec{J} - 30\vec{K}}{\underbrace{\sqrt{10^2 + 15^2 + 30^2}}_{35}} = 0.286\vec{I} + 0.429\vec{J} - 0.857\vec{K}$$

$$\underline{\vec{F_C} = 14.3\vec{I} + 21.5\vec{J} + (-42.9)\vec{K}}\ (lb)$$

2- MOMENT AT C

$$\vec{M_C} = \vec{CA} \times \vec{F} = \begin{vmatrix} \vec{I} & \vec{J} & \vec{K} \\ 0 & 30 & 30 \\ 14.3 & 21.5 & -42.9 \end{vmatrix} = -1932\vec{I} + 429\vec{J} - 429\vec{K}$$

$$\underline{\vec{M_A} = -1932\vec{I} + 429\vec{J} - 429\vec{K}}\ (lb\text{-}FT)$$

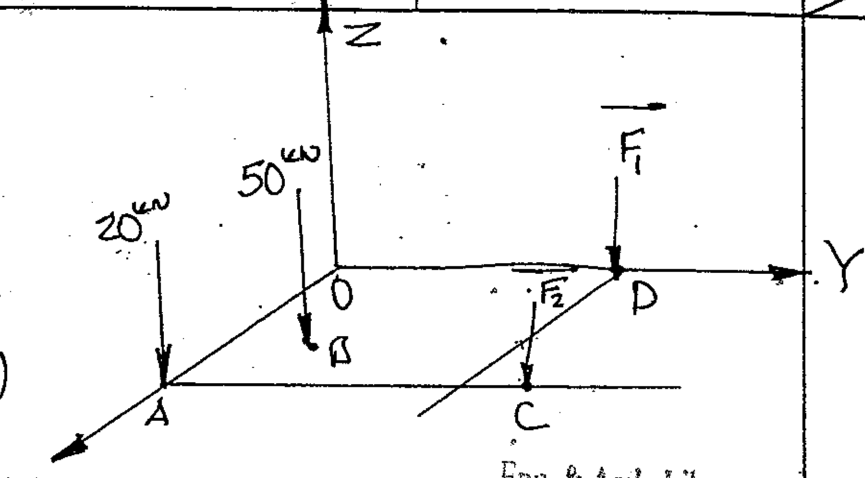
PROBLEM 4-134

$F_1 = 20^{kN}$

$F_2 = 50^{kN}$

$A(10,0,0)$ $B(4,3,0)$

$C(10,13,0)$ $D(0,11,0)$



EQUIVALENT RESULTANT FORCE

$$\overrightarrow{F_R} = \overrightarrow{\Sigma F} \overset{+}{=} \left(20^{kN} + 50^{kN} + F_1 + F_2\right)\overrightarrow{K}$$

$$\underline{\overrightarrow{F_R} = +140\,K}$$

LOCATION OF RESULTANT ASSUME RESULTANT AT (X_R, Y_R)

$$\overrightarrow{M_R} = \Sigma \vec{r} \times \vec{F}, \quad \overrightarrow{M_R} = \left(X_R\vec{I} + Y_R\vec{J}\right) \times \overrightarrow{F_R} = \vec{0}$$

$$\overrightarrow{M_R} = \left(X_R\vec{I} + Y_R\vec{J}\right) \times \left(+140\vec{K}\right) \Rightarrow \left(+140X_R\right)\vec{J} - 140Y_R\vec{I} = 0$$

$$\Sigma\vec{r}\times\vec{F} = \overrightarrow{OA}\times\left(-\overrightarrow{20K}\right) + \overrightarrow{OB}\times\left(-\overrightarrow{50K}\right) + \overrightarrow{OD}\times\left(-\overrightarrow{20K}\right) + \overrightarrow{OC}\times\left(-\overrightarrow{50K}\right)$$

$$= \left[\left(-200\vec{I}\right) + \left(-200\vec{I} - 150\vec{J}\right) + \left(-220\vec{J}\right) + \left(-500\vec{I} - 650\vec{J}\right)\right] \times \vec{K}$$

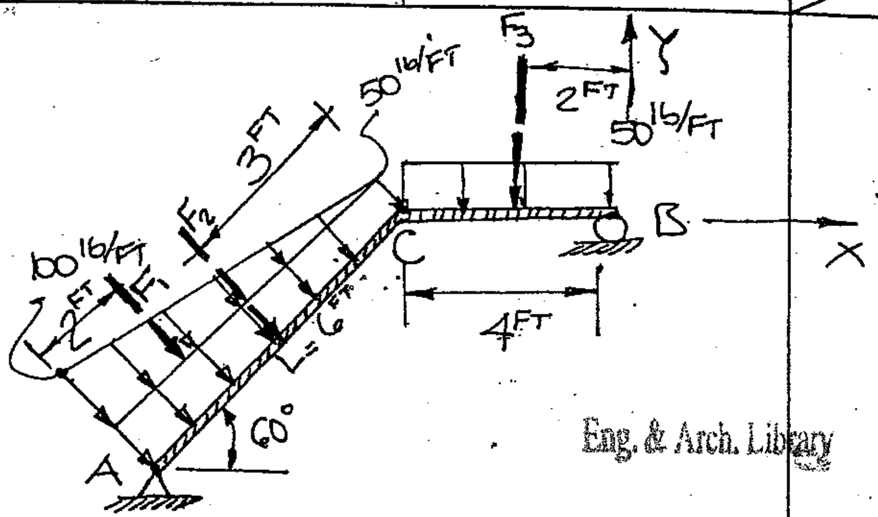
$$= \left(-900\vec{I} - 1020\vec{J}\right) \times \vec{K}$$

$$= +900\vec{J} - 1020\vec{I}$$

THEREFORE: $+140X_R = +900$ $\qquad \underline{X_R = 6.43^M}$

$\&$ $-140Y_R = -1020$ $\qquad \underline{Y_R = 7.29^M}$

PROBLEM 4-153



F_1 RESULTANT OF TRIANGULAR LOAD:

$F_1 = 50^{lb/FT} * \frac{6^{FT}}{2} = 150^{lb}$ NORMAL TO AC, 2^{FT} FROM A

$F_{1X} = F_1 \cos 30 = 129.9^{lb}$

$F_{1Y} = -F_1 \cos 60 = -75^{lb}$

F_2 RESULTANT OF RECTANGULAR LOAD:

$F_2 = 50^{lb/FT} * 6^{FT} = 300^{lb}$ NORMAL TO AC, 3^{FT} FROM A

$F_{2X} = 300 \cos 30 = 259.8^{lb}$

$F_{2Y} = -300 \cos 60 = -150^{lb}$

F_3 RESULTANT OF HORIZONTAL RECTANGULAR LOAD:

$F_3 = 50^{lb/FT} * 4 = 200^{lb}$

$F_{3X} = 0 \qquad F_{3Y} = -200^{lb}$

EQUIVALENT SYSTEM @ C:

$F_{RX}^{C} = F_{1X} + F_{2X} + F_{3X} = 129.9 + 259.8 + 0 = 389.7^{lb}$

$F_{RY}^{C} = F_{1Y} + F_{2Y} + F_{3Y} = -75 - 150 - 200 = -425^{lb}$

$M_R^C = F_1 * 4 + F_2 * 3 - F_3 * 2 = 4 * 150 + 3 * 300 - 2 * 200 = 1100^{lb\text{-}FT}$ or $\odot$

EQUIVALENT SYSTEM @ B:

$$F_{RX}^{B} = F_{RX}^{C} = 389.7^{lb} \longrightarrow = F_{RX}^{B}$$

$$F_{RY}^{B} = F_{RY}^{C} = 425^{lb} \downarrow = F_{RY}^{B}$$

$$F_{R}^{B} = \sqrt{389.7^2 + 425^2} = 577^{lb} = F_{R}^{B}$$

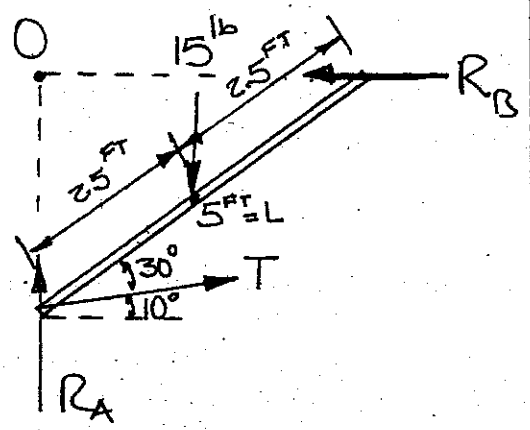
$$\text{ANGLE} = \tan^{-1}\left(\frac{425}{389.7}\right) = 47.5^{\circ} = \text{ANGLE}$$

$$M_{R}^{B} = M_{R}^{C} + F_{RY}^{C} * 4 = 1100^{lb\text{-}FT} + 4 * 425^{lb\text{-}FT} = 2800^{lb\text{-}FT} = M_{R}^{B} \odot \text{ or } \circlearrowright$$

PROBLEM 5_32

REACTION @ B = HORIZONTAL

REACTION @ A = VERTICAL



FBD OF AB

EQUILIBRIUM ABOUT POINT O

$$\Sigma M_O = 0 = M_{WEIGHT} + M_T$$

$$0 = 15^{lb} * (2.5 \cos 40)^{FT} - (T \cos 10) * (5 \cos 50)^{FT}$$

$$0 = 28.73 - T * 3.165$$

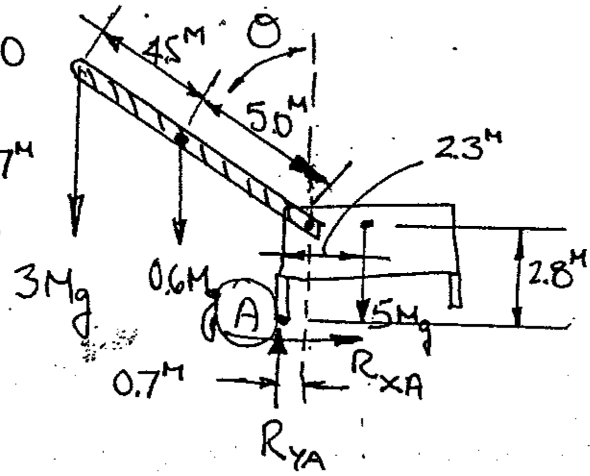
$$T = 9.08^{lb}$$

PROBLEM 5-38

MAXIMUM BOOM ANGLE θ BEFORE OVERTURN $\longrightarrow$ REACTION AT B = 0

MOMENT ARM OF $3^{Mg} = 9.5^{M}\sin\theta - 0.7^{M}$

MOMENT ARM OF $0.6^{Mg} = 0.6^{M}\sin\theta - 0.7$



EQUILIBRIUM ABOUT POINT A:

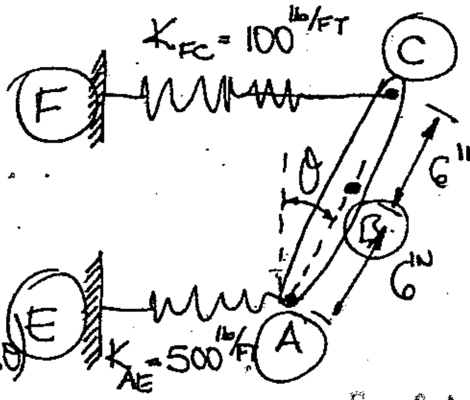
$$\sum M_A = 0 \longrightarrow 3^{Mg} * (9.5^{M}\sin\theta - 0.7) + 0.6^{Mg}(5^{M}\sin\theta - 0.7) - 5^{Mg} * 2.3^{M} = 0$$

(+)

$$\longrightarrow (28.5 + 3)\sin\theta = 14.02$$

$$\sin\theta = 0.445 \qquad \underline{\theta = 26.4^{\circ}}$$

PROBLEM 5_52



MOMENT EQUILIBRIUM ABOUT B:

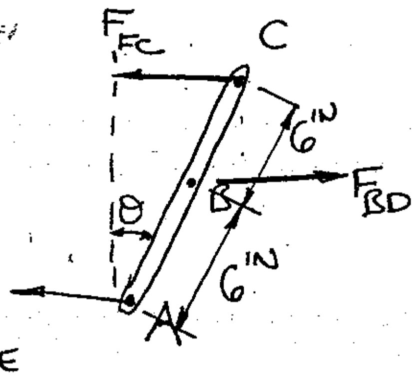
$$\Sigma M_B = 0 = M_{FC} + M_{AE}$$
$$= F_{FC}(6\cos\theta) - F_{AE}(6\cos\theta)$$

$$\longrightarrow F_{FC} = F_{AE}$$

$$F_{FC} = K_{FC}\Delta_{FC} = 100\,\Delta_{FC}$$

$$F_{AE} = K_{AE}\,\Delta_{AE} = 500\,\Delta_{AE}$$

THEREFORE, $\Delta_{FC} = 5\Delta_{AE}$ (1)



FBD

ELONGATION RELATIONSHIP

Δ_{FC} = ELONGATION OF SPRING FC

Δ_{AE} = ELONGATION OF SPRING AE

$$\frac{1}{2}(\Delta_{FC} + \Delta_{AE}) = 2^{IN} \quad (2)$$

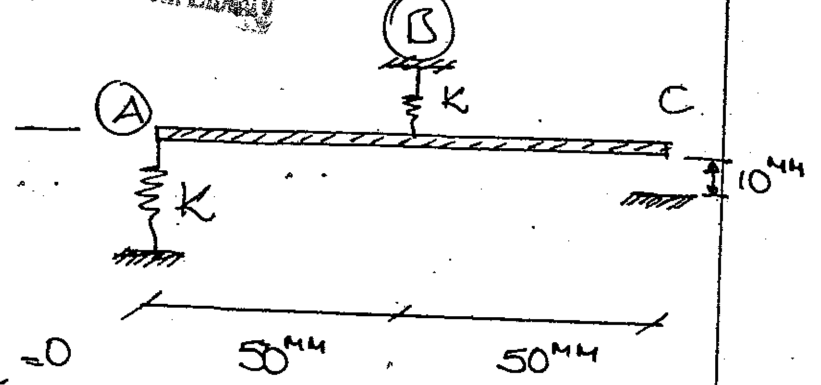
(1) & (2) $\longrightarrow \frac{1}{2}(6\Delta_{AE}) = 2^{IN}, \quad \Delta_{AE} = \frac{2}{3}^{IN}$

$$\Delta_{FC} = \frac{10}{3}^{IN}$$

$$\theta = \sin^{-1}\left(\frac{\Delta_{FC} - \Delta_{AE}}{12}\right) = \sin^{-1}\left(\frac{8/3^{IN}}{12}\right)$$

$$\underline{\theta = 12.84^\circ}$$

PROBLEM 5-56



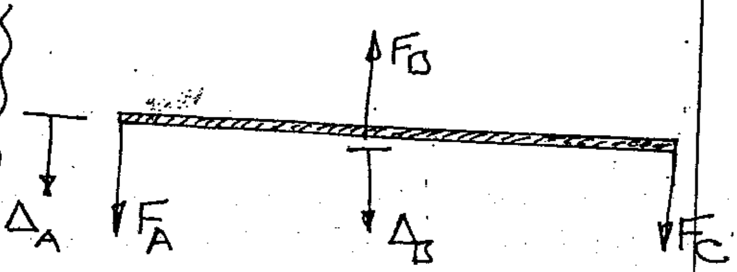
FORCE AT C = 0.5^N

EQUILIBRIUM ABOUT B

$$\Sigma M_B = 0 \longrightarrow 50F_A - 50F_C = 0$$

$0.5^N = F_A = F_C$, $K = \dfrac{0.5^N}{\Delta_A}$

Δ_A = DISPLACEMENT AT A ↓
= SHORTENING OF SPRING AT A



VERTICAL EQUILIBRIUM

$$F_B = F_A + F_C = 1.0^N \qquad K = \frac{1.0^N}{\Delta_B}$$

F_B TENSION

Δ_B = DISPLACEMENT AT B ↓
= ELONGATION OF SPRING AT B

ELONGATION EQUATION

$$\frac{1}{2}\left(10^{MM} + \Delta_A\right) = \Delta_B$$

$$\frac{1}{2}\left(10^{MM} + \frac{0.5^N}{K^{N/MM}}\right) = \frac{1.0^N}{K^{N/MM}} \longrightarrow 5 + \frac{0.25}{K} = \frac{1}{K}$$

$$\longrightarrow K = \frac{0.75}{5}^{N/MM}$$

$$\underline{K = 0.15^{N/MM}}$$